respect to m.

Taking as basic bodies with exponents $m_1 = 0.5$, $m_2 = 0.6$, $m_3 = 0.7$, for $m_0 = 0.55$ and $m_0 = 0.65$ and using formula (4) we obtain, $\beta_1 = 0.374$, $\beta_2 = 0.758$, $\beta_3 = -0.134$ and $\beta_1 = -0.121$, $\beta_2 = 0.734$, $\beta_3 = 0.389$ respectively.

Curves of the functions $C_x(\lambda/m)$ are shown in Fig.1, calculated using the results obtained in /6/ for a Mach number $M_\infty = 2.6, \infty; \gamma = 1.4$ of the oncoming flow, and those calculated using the method described in /7/ are presented in Fig.2 for bodies of exponential form with a spherical nose (this deformation does not require a recalculation of β_ν) for three-dimensional flow of an ideal gas over a body $\alpha = 10°$, $M_\infty = 20$, $\gamma = 1.4$. The small circles and dots correspond to recalculations using Eq.(3), the solid lines correspond to $m_0 = 0.65$, and the dashed lines to $m_0 = 0.55$.

It can be seen that the determination of aerodynamic calculations using relations (3) enables us to obtain estimates of the aerodynamics force components that are very close to the exact calculations for the supersonic flow of an ideal gas over a body. This result applies also to the flow of an equilibrium and non-equilibrium dissociating gas. For actual gas flows the non-dependence of the coefficients β_ν on the flow conditions over the body, the angle of attack and, also, of which aerodynamic force component is considered, is confirmed.

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EQUILIBRIUM IN A CUT ALONG AN ARC OF A CIRCLE IN THE CASE OF INHOMOGENEOUS INTERACTION OF THE EDGES*

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Equilibrium in a cut along the arc of a circle is considered for the case of biaxial tension-compression. Under such a stress a free surface forms along the cut, and a zone of adhesion and mutual displacement appears in the region of contact when frictional forces are present. A non-singular solution is constructed for this case at the boundary of the zone of contact and free surface, and of the zone of adhesion and mutual displacement. Earlier, the problems of the free surface as well as the region of contact were considered in /1–3/. A solution was found in /4/ for a cut along the arc of a circle in a complex state of stress for the case when the edges interact at the extension of the cut, taking into account the formation of the adhesion and displacement zones.

1. Consider a cut along the arc of a unit circle. The equation of the cut in x, y-coordinates is $x = \cos\theta$, $y = \sin\theta$, $\alpha_0 \leqslant \theta \leqslant \beta_0$ (α_0, β_0 are the coordinates of the cut boundary). We have at infinity the mutually perpendicular stresses p, q ($p \leqslant 0, q \geqslant 0$) and p makes an angle γ with the Ox-axis. We shall describe the stress state using the complex Kolosov-Muskhelishvili potentials $\Phi(z)$, $\Psi(z)$ /1/

$$\sigma_r + \sigma_\theta = 2[\Phi(z) + \Phi^*(z)] \tag{1.1}$$

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$$\sigma_r + i\tau_{r\theta} = \Phi(z) + \Omega\left(\frac{1}{z^*}\right) + z^*\left(z^* - \frac{1}{z}\right)\Psi^*(z)$$
$$2G(u' + iv') = iz\left[\gamma\Phi(z) - \Omega\left(\frac{1}{z^*}\right) - z^*\left(z^* - \frac{1}{z}\right)\Psi^*(z)\right]$$
$$\Omega(z) = \Phi^*\left(\frac{1}{z^*}\right) - \frac{1}{z}\Phi^*\left(\frac{1}{z^*}\right) - \frac{1}{z^2}\Psi^*\left(\frac{1}{z^*}\right)$$
$$u' = \frac{\partial u}{\partial\theta}, \quad v' = \frac{\partial v}{\partial\theta}$$

Here $\varkappa = 3-4\nu$ for plane deformation, $\varkappa = (3-\nu)/(1+\nu)$ for the generalized plane state of stress, ν is Poisson's ratio, G is the shear modulus, u and v are the displacement components along the Ox and Oy axes respectively and $\sigma_r, \sigma_\theta, \tau_{r\theta}$ are the stress tensor components in a system of polar coordinates with origin at the point O. The displacement components in rectangular and polar coordinate v_r, v_θ systems are connected by the following relations:

$$u + iv = (v_r + iv_\theta)e^{i\theta} \tag{1.2}$$

The potentials $\Phi(z), \Omega(z)$ must be connected by the relation /1/

$$\Phi(0) = \Omega^*(\infty) \tag{1.3}$$

and the complex potentials have the following asymptotic forms:

$$|z| \to \infty, \quad \Phi(z) = \Gamma, \ \Psi(z) = \Gamma', \ \Omega(z) = -\Gamma^{*\prime}/z^2 \tag{1.4}$$
$$\Gamma = (p+q)/4, \ \Gamma' = -{}^1/_2(p-q)e^{-2i\gamma}, \quad |p| \geqslant |q|$$

We shall assume that only a single region of the free surface appears along the cut, and we will denote it by L_1, (e.g. at $0 \leqslant \theta - \gamma \leqslant \pi$). When there are no frictional forces the region of mutual displacements (we shall denote it by L) occupies the whole of the cut, while when there are frictional forces present a zone of mutual shear displacements (denoted by L_2) adjacent, in general, to the zone of adhesion, will be adjacent to the free surface. Since there are no mutual displacements in the adhesion zone, we shall regard it as a continuum and specify the shear boundary condition in the domain of mutual displacements.

2. Let us consider the case when there are no frictional forces from the region of contact between the cut edges. In this case the boundary conditions at the cut will be

$$\tau_{r\theta}^{\pm} = 0, \ r \to R \pm 0; \ t \in L, \ \sigma_r^{\pm} = 0, \ t = L_1 \tag{2.1}$$
$$v_r^+ - v_r^- = 0, \ t \in L_2 \quad (t = e^{i\theta}, \ \alpha_0 \leqslant \theta \leqslant \beta_0)$$

Using the Kolosov-Muskhelishvili relations /1/ and conditions (2.1), we arrive at the following conjugation problem:

$$\Phi^+ + \Phi^- - (\Phi^{*+} + \Phi^{*-}) + \Omega^+ + \Omega^- - (\Omega^{*+} + \Omega^{*-}) = 0, \ t \in L \tag{2.2}$$
$$\Phi^+ + \Phi^- + \Phi^{*+} + \Phi^{*-} + \Omega^+ + \Omega^- + \Omega^{*+} + \Omega^{*-} = 0, \quad t \in L_1$$
$$\Phi^+ - \Phi^- - (\Omega^+ - \Omega^-) = 0, \ t \in L$$
$$2G[(u'^+ - u'^-) + i(v'^+ - v'^-)] = it[\varkappa(\Phi^+ - \Phi^-) + \Omega^+ - \Omega^-], \quad t \in L$$

We shall seek a solution of (2.2) in the form /3, 5/

$$\Phi(z) = \frac{1}{2\pi}\int_L \frac{\eta(t)\,dt}{t-z} + \Gamma \tag{2.3}$$
$$\Omega(z) = \frac{1}{2\pi}\int_L \frac{\eta(t)\,dt}{t-z} + \Gamma - \frac{\Gamma'}{z^2} - D_0, \quad t \in L$$

where (taking (1.2) and (1.3) into account)

$$\eta = -\frac{2G}{\varkappa+1}\,\frac{(u'^+ - u'^-) + i(v'^+ - v'^-)}{t} = -(g_1' + ig_1 - g + ig'), \tag{2.4}$$
$$g(\theta) = \frac{2G}{\varkappa+1}(v_\theta^+ - v_\theta^-)$$
$$g_1(\theta) = \frac{2G}{\varkappa+1}(v_r^+ - v_r^-)$$
$$D_0 = \frac{1}{2\pi}\int \frac{\eta^*\,dt}{t} = \frac{1}{2\pi}\left[i\int_L g\,d\theta - \int_{L_2} g_1\,d\theta\right]$$

The integrals in expressions (2.3) are computed as $t \to t_0 \in L$ using the Sokhotskii-Plemilj /6/ formulas. On substituting expressions (2.3) for the complex potentials $\Phi(z), \Omega(z)$, we find that the last two equations will be satisfied identically and the first equation will be reduced to a singular integral equation with a solution of the form /6/

$$\eta - \eta^* = iA(t)/x(t) \tag{2.5}$$

$$A(t) = A_1 t^3 + A_2 t^2 + A_3 t + A_4 + \frac{D_1}{t} + \frac{D_2}{t^2} + C$$

$$A_1 = -\Gamma', \quad A_2 = -A_1\frac{a+b}{2}, \quad A_3 = -\frac{(a-b)^2}{8}A_1 + B,$$

$$A_4 = -B\frac{a+b}{2} - \frac{(a+b)(a-b)^2}{16}A_1, \quad D_1 = \frac{a+b}{2\sqrt{ab}}\Gamma'^*,$$

$$D_2 = -\Gamma'^*\sqrt{ab}, \quad B = -(D_0 + D_0^*)$$

Multiplying both sides of (2.5) by $1/t\ (t \in L)$ and integrating along the cut, we obtain the relation

$$C = -A_4 + A_3^*\sqrt{ab} \tag{2.6}$$

Substituting the expression for the function $\eta(t)$ (2.4) and using relation (2.6), we arrive at a differential equation whose solution has the form

$$h = 0,\ t \in L_2;\ h = 1,\ t \in L_1 \tag{2.7}$$

$$g(\theta) = \left(F_1 t + F_2 + \frac{F_3}{t} + \frac{F_4}{t^2}\right)\frac{X(t)}{2} + s + h\int_\theta^{\beta_1} g_1(\theta)\,d\theta$$

$$F_1 = -\frac{A_1}{2i}, \quad F_2 = F_1\frac{a+b}{2}, \quad F_3 = F_4\frac{a+b}{2ab}, \quad F_4 = F_1^*/\sqrt{ab}$$

$$s = -Bf_1, \quad f_1 = \frac{\pi}{2} - \arcsin\left(\sin\left(\frac{\theta}{2} - \frac{\alpha+\beta}{4}\right)\sin^{-1}\left(\frac{\beta-\alpha}{4}\right)\right)$$

where α_1, β_1 denote the unknown boundaries of the free surface and $(\alpha_1 \leqslant \beta_1)$.

Substituting (2.3) into the second equation of (2.2), we arrive at a singular integral equation whose solution, utilizing the Poincare-Bertrand inversion formula /6/, has the form

$$g_1 - \int_\theta^{\beta_1} g_1\,d\theta = \frac{iR(t)}{x_1(t)} + s(t) - \frac{i}{\pi x_1}\int_{L_2}\frac{s x_1\,dt_1}{t_1 - t} - \frac{i}{\pi x_1}\int_L s\,dt, \quad t \in L_1 \tag{2.8}$$

$$R(t) = R_1 t^3 + R_2 t^2 + R_3 t + R_4 + \frac{M_1}{t} + \frac{M_2}{t^2} + C_1$$

$$R_1 = -\frac{3\Gamma'}{4}, \quad R_2 = -R_1\frac{d+f}{2}, \quad R_3 = -\frac{(d-f)^2}{8}R_1 + B_1$$

$$R_4 = -\left(R_1\frac{(d-f)^2}{8} + B_1\right)\frac{d+f}{2}$$

$$B_1 = \frac{\Gamma'^*(a+b)}{8ab\sqrt{ab}} + \frac{\Gamma'(3a^2 + 2ab + 3b^2)}{32} - \frac{3D_0}{2} - \frac{D_0^*}{2} + 2\Gamma$$

$$M_1 = -\frac{(d+f)3\Gamma'^*}{8\sqrt{df}}, \quad M_2 = \frac{3\Gamma'^*}{4}\sqrt{df}; \quad x_1 = \sqrt{(z-d)(z-f)}$$

$$d = e^{i\alpha_1}, \ f = e^{i\beta_1}$$

Multiplying both sides of (2.8) by $1/t\ (t \in L_1)$ and integrating over L_1, we obtain

$$C_1 = -R_4 - R_3^*\sqrt{df} - \frac{1}{\pi}\int_{L_2}\frac{s x_1}{t}\,dt + \frac{1}{\pi}\int_L s\,dt \tag{2.9}$$

An unknown constant D_0 appears in (2.7) and (2.8). To determine this constant we note that the constants C and C_1 are connected by the relation

$$C_1 = \frac{C}{2} + \frac{1}{2\pi}\int_L\left(F_1 t + F_2 + \frac{F_3}{t} + \frac{F_4}{t^2}\right)x(t)\,dt + \frac{1}{\pi}\int_L s\,dt \tag{2.10}$$

Using (2.6), (2.9), (2.10) we obtain

$$D_0 = (z_1^* l - z_1 k^*)/(ll^* - kk^*) \tag{2.11}$$

$$z_1 = \frac{\Gamma'}{16}((3a^2 + 2ab + 3b^2)(d+f) - 8(a+b)\sqrt{ab}\sqrt{df} -$$

$$3(d+f)(d-f)^2) + \frac{\Gamma'^*}{8}\left(\frac{2(d+f)(a+b)}{ab\sqrt{ab}} - \frac{3(d-f)^2}{df\sqrt{df}} -\right.$$

$$\left.\frac{(3a^2 + 2ab + 3b^2)}{a^2b^2}\sqrt{df}\right) + 4\Gamma(\sqrt{d} - \sqrt{f})^2, \quad k = 3(d+f) - 2\sqrt{df} - r_1$$

$$l = d + f - 6\sqrt{df} - r_1, \quad r_1 = -\frac{4}{\pi}\int_L\frac{f_1 x_1}{t}\,dt + (\sqrt{a} + \sqrt{b})^2$$

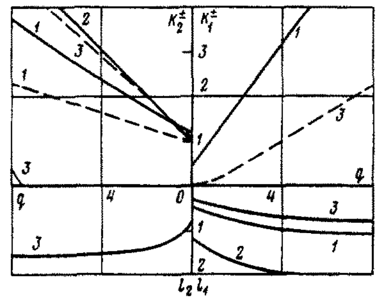
In general, expressions (2.8), taking (2.10) and (2.11) into account, define a singular solution at the points $t = d,\ t = f$, characterized by the following intensity coefficients /5/:

$$K_1^+ = \lim_{\theta\to\beta_1}\sqrt{2(\beta_1 - \theta)}\,|g_1'(\theta)|; \quad K_1^- = \lim_{\theta\to\alpha_1}\sqrt{2(\theta - \alpha_1)}\,|g_1'(\theta)| \tag{2.12}$$

Since the singular normal stresses at the cut near the boundary of the region of the free surface will always result in coupling on the compressed section of the crack, we shall construct a non-singular solution at this boundary. By analogy with the criterion used in /2, 3/ we shall use, as the condition of equilibrium, that the stress intensity coefficients equal zero

$$R(t) = 0 \tag{2.13}$$

Equation (2.13) determines the unknown boundaries of the free surface and the contact zone. If both roots of the equation are found to lie inside the cut ($\alpha_0 \leqslant \alpha_1 \leqslant \beta_1 \leqslant \beta_0$), then the zone of contact touches the left and right tip of the initial cut, while if one of the roots lies outside the cut, free-surface zone appears, touching one of the crack tips. Thus Eq. (2.12) and expressions (2.7), (2.8), (2.10), (2.11) together determine the stress state of a solid, weakened by a plane, arc-like cut.



The figure shows a graph of the stress intensity coefficients $K_1^+, K_1^-, K_2^+, K_2^-$ (the solid lines show the quantities with the plus index, and the dashed lines those with the minus index), at the crack tip and the length of the region of free surface on the stresses p, q $(p = -5)$ in the case when the cut is situated in the region $\pi/2 \leqslant \theta \leqslant \pi$, $\gamma = 0$ (curves *1*) and $0 \leqslant \theta \leqslant \pi, \gamma = 0$ (curves *2*) for $\mu = 0.4$.

3. Let us now consider the case when frictional forces exist between the crack edges in the region of contact. We shall assume that the arc-like cut is situated in a region, along which we have a single region of mutual displacements along which the shear stresses do not change their sign (e.g. $\pi/2 \leqslant \theta - \gamma \leqslant \pi$). In this case the boundary conditions have the form

$$\begin{gathered}\tau_{r\theta}^{\pm} = \rho\sigma_r^{\pm},\ r \to R \pm 0,\ t \in L_2,\ \sigma_r^{\pm} = 0,\ t \in L_1 \\ v_r^+ - v_r^- = 0,\ t \in L_2\ (t = e^{i\theta},\ \alpha_0 \leqslant \alpha_2 \leqslant \theta \leqslant \beta_2 \leqslant \beta_0)\end{gathered} \tag{3.1}$$

where $\rho = \pm\mu$, μ is the coefficient of friction and the plus or minus sign in determined by the direction of shear stresses appearing at the cut site in the solid (e.g. when $\pi/2 \leqslant \theta - \gamma \leqslant \pi$, we have $\tau_{r\theta}^{\pm} = \mu\sigma_r^{\pm}$, and for $0 \leqslant \theta - \gamma \leqslant \pi/2$ we have $\tau_{r\theta}^{\pm} = -\mu\sigma_r^{\pm}$), α_2, β_2 are so far unknown boundaries of the region of mutual displacements.

Using relations (1.1) and (3.1), we arrive at the following conjugate problem:

$$\begin{gathered}(\Phi^+ + \Phi^-)(\rho + i) + (\Phi^{+*} + \Phi^{-*})(\rho - i) + (\Omega^+ + \Omega^-)(\rho + i) + \\ (\Omega^{+*} + \Omega^{-*})(\rho - i) = 0,\ t \in L \\ \Phi^+ + \Phi^- + \Phi^{+*} + \Phi^{-*} + \Omega^{+*} + \Omega^{-*} + \Omega^+ + \Omega^- = 0,\ t \in L_1 \\ \Phi^+ - \Phi^- - (\Omega^+ - \Omega^-) = 0,\ t \in L \\ 2G\left[(u'^+ - u'^-) + i(v'^+ - v'^-)\right] = it\left(\varkappa(\Phi^+ - \Phi^-) + (\Omega^+ - \Omega^-)\right),\quad t \in L\end{gathered} \tag{3.2}$$

We seek the solution of (3.2) in the form (2.3). Substituting the latter into the first equation of (3.2) we obtain the following singular equation /6/:

$$\int_L \frac{\eta(\rho + i) + \eta^*(\rho - i)}{t - t_0}\,dt = \pi p(t_0),\quad p(t) = \frac{\Gamma'^*}{t^2}(\rho + i) + \Gamma' t^2(\rho - i) - 4\Gamma\rho + D_0(3\rho - i) + D_0^*(\rho - i)$$

The solution of this equation has the form /6/

$$\begin{gathered}\eta(\rho + i) + \eta^*(\rho - i) = -\frac{1}{\pi x(t_0)}\int_L \frac{x(t)\,p(t)\,dt}{t - t_0} + \frac{Ci}{x(t_0)} \\ C = (1 - i\rho)\int_L \eta^* dt,\ x(t) = \sqrt{(t - a)(t - b)},\ a = e^{i\alpha_2},\ b = e^{i\beta_2}\end{gathered} \tag{3.3}$$

Computing the integral on the right-hand side of (3.3), we obtain

$$\begin{gathered}\eta(\rho + i) + \eta^*(\rho - i) = i\,\frac{A(t)}{x(t)} \\ A(t) = A_1t^3 + A_2t^2 + A_3t + A_4 + C + \frac{D_1}{t} + \frac{D_2}{t^2} \\ A_1 = \Gamma'(\rho - i),\quad A_2 = -A_1\frac{(a + b)}{2},\quad A_3 = -\frac{(a - b)^2 A_1}{8} + B \\ B = D_0(3\rho - i) + D_0^*(\rho - i) + 4\Gamma\rho,\quad A_4 = -B\frac{a + b}{2} - \frac{(a + b)(a - b)^2}{16}A_1,\quad D_1 = -A_2^*\sqrt{ab},\quad D_2 = -A_1^*\sqrt{ab}\end{gathered} \tag{3.4}$$

An expression for C is obtained in the same way as (2.6)

$$C = -A_4 - A_3^*\sqrt{ab} \tag{3.5}$$

Substituting the expression for the function $\eta(t)$ into (3.4) and using (3.5), we arrive at a differential equation whose solution has the form

$$h = 0,\ t \in L_2;\ h = 1,\ t \in L_1 \tag{3.6}$$

$$g(\theta) = \left(F_1 t + F_2 + \frac{F_3}{t} + \frac{F_4}{t^2}\right) \frac{x(t)}{2} + s(t) + he^{-\rho\theta} \int_\theta^{\beta_1} (g_1 - \rho g_1') e^{\rho\theta} d\theta$$

$$F_1 = \frac{iA_1}{\rho + 2i}, \quad F_2 = \frac{iA_1(a+b)}{2(\rho + 2i)}, \quad F_3 = \frac{F_2^*}{\sqrt{ab}}, \quad F_4 = \frac{F_1^*}{\sqrt{ab}}$$

$$s(t) = e^{-\rho\theta} \int_\theta^{\beta_1} \frac{e^{\rho\theta}(K_1 t + K_2)}{2x(t)} d\theta$$

$$K_1 = iA_3 - abF_1(\rho + i) + F_2(a+b)\left(\rho + \frac{i}{2}\right) - F_3\rho, \quad K_2 = \frac{K_1^*}{\sqrt{ab}}$$

where α_1, β_1 are the boundaries of the free surface. Substituting expression (2.3) into the second equation of (3.2) and using the solution (3.6), we arrive at a singular integral equation whose solution, like (2.8), has the form

$$g_1' - e^{-\rho\theta} \int_\theta^{\beta_1} e^{\rho\theta}(g_1 - \rho g_1') d\theta = \frac{iR(t)}{x_1(t)} + s(t) - \frac{i}{\pi x_1(t)} \int_{L_2} \frac{t x_1 s \, dt_1}{t_1(t_1 - t)}, \quad t \in L_1, \quad R(t) = R_1 t^3 + R_2 t^2 + R_3 t - \tag{3.7}$$

$$R_3^* \sqrt{df} + \frac{M_1}{t} + \frac{M_2}{t^2}, \quad R_1 = -\frac{3\Gamma' i}{2(\rho + 2i)}$$

$$R_2 = -R_1 \frac{d+f}{2}, \quad R_3 = -R_1 \frac{(d-f)^2}{8} + B_1,$$

$$R_4 = \left(-R_1 \frac{(d-f)^2}{8} - B_1\right) \frac{d+f}{2}, \quad M_1 = -R_2^* \sqrt{df},$$

$$B_1 = -\frac{\Gamma'^*(\rho + i)(a+b)}{4ab\sqrt{ab}(\rho - 2i)} - \frac{\Gamma'(\rho - i)}{\rho + 2i} \frac{(3a^2 + 2ab + 3b^2)}{16} - \frac{3D_0}{2} - \frac{D_0^*}{2} + 2\Gamma$$

$$M_2 = -R_1^* \sqrt{df}, \quad x_1(z) = \sqrt{(z-d)(z-f)}, \quad d = e^{i\beta_1}, \quad f = e^{i\alpha_1}$$

In determining the constant D_0 we note that C and C_1 are connected by the relation

$$C_1 = -\frac{C}{2(\rho - i)} + \frac{1}{2\pi} \int_L \left(F_1 t + F_2 + \frac{F_3}{t} + \frac{F_4}{t^2}\right) x(t) dt + \frac{1}{\pi} \int s \, dt \tag{3.8}$$

Using (3.5),(3.6) and (3.8) we obtain

$$D_0 = (z_1^* l - z_1 k^*)/(ll^* - kk^*) \tag{3.9}$$

$$z_1 = \frac{\Gamma'}{8}\left(-\frac{3i(d+f)(d-f)^2}{(\rho + 2i)} - \frac{(\rho - i)}{(\rho + 2i)}(3a^2 + 2ab + 3b^2)(d+f) + \frac{8(\rho - i)(a+b)\sqrt{ab}\sqrt{df}}{(\rho + 2i)} + \frac{3(a+b)(a-b)^2\rho}{(\rho + 2i)}\right) +$$
$$\frac{\Gamma'^*}{4}\left(-\frac{2(a+b)(d+f)(\rho + i)}{ab\sqrt{ab}(\rho - 2i)} - \frac{3i(d-f)^2}{(\rho - 2i)\sqrt{df}\, df} + \frac{(\rho + i)}{\rho - 2i} \frac{(3a^2 + 2ab + 3b^2)}{a^2 b^2} \sqrt{df} - \frac{(a-b)^2(\rho + i)\rho}{ab\sqrt{ab}(\rho - i)(\rho - 2i)}\right) +$$
$$4\Gamma(\sqrt{d} - \sqrt{f})^2 + \frac{\Gamma\rho(\sqrt{a} - \sqrt{b})^2}{\rho - i} + \frac{1}{\pi} \int_{L_2} \frac{(s_1 G_1 + s_1^* G_1^*)}{t} x(t) dt$$

$$k = 3(d+f) - 2\sqrt{df} - \frac{(3\rho - i)}{\rho - i}(a+b) + \frac{2\sqrt{ab}(\rho + i)}{\rho - i} + Q_1$$

$$l = d + f - 6\sqrt{df} + \frac{2(3\rho + i)}{\rho - i}\sqrt{ab} - (a+b) + Q_2$$

$$s_1 = e^{\rho\theta} \int_\theta^{\beta_2} \frac{e^{\rho\theta'} t}{x(t)} dt, \quad G_1 = -\frac{i(a-b)^2}{8} A_1 - abF_1(\rho + i) + F_2(a+b)\left(\rho + \frac{i}{2}\right) - F_3\rho,$$

$$Q_1 = i \int_{L_2} \frac{(c_1(3\rho - i) - c_1^*(\rho + i))}{t} x(t) dt, \qquad Q_2 = -i \int_{L_2} \frac{(c_1(\rho - i) - (3\rho + i)c_1^*)}{t} x(t) dt$$

The expressions (3.6), (3.7), (3.9) determine, in general, the singular solution at the point $\theta = \beta_1, \theta = \alpha_1, \theta = \beta_2, \theta = \alpha_2$, characterized by the stress intensity coefficients /5/

$$K_1^- = \lim_{\theta \to \alpha_1} \sqrt{2(\theta - \alpha_1)} \, |g'(\theta)|, \quad K_2^- = \lim_{\theta \to \alpha_2} \sqrt{2(\theta - \alpha_2)} \, |g_1'(\theta)| \tag{3.10}$$
$$K_1^+ = \lim_{\theta \to \beta_1} \sqrt{2(\beta_1 - \theta)} \, |g'(\theta)|, \quad K_2^+ = \lim_{\theta \to \beta_2} \sqrt{2(\beta_2 - \theta)} \, |g_1'(\theta)|$$

The boundary of the free surface and region of contact are defined by the equation (2.13) Since the singular shear stresses at the cut in the neighbourhood of the boundary of the region of mutual displacements will always result in coupling of the compressed segment of the crack, therefore we construct a non-singular solution at the boundary separating the adhesion zone from the mutual displacement zone. As in /4/, the condition of equilibrium will be obtained by equating to zero the stress intensity coefficients

$$A(t) = 0 \tag{3.11}$$

Equation (3.11), taking (3.5) into account, gives the unknown boundaries of the adhesion and mutual displacement zones. If both roots of this equation lie within the cut $(\alpha_0 \leqslant \alpha_2 \leqslant \beta_2 \leqslant \beta_0)$, then the solution constructed will be singular in the neighbourhood of one tip, and the adhesion zone will be in contact with the other tip.

As an example, consider a cut situated in the region $\pi \leqslant \theta \leqslant \pi/2$, $\gamma = 0$. In this case we have $\rho = \mu$ in Eq.(3.1). The figure shows the relation connecting the stress intensity coefficients and the length of the free surface domain with the stress (the solid and dashed lines correspond to the right and left crack tip respectively) when $\mu = 0.4$, $p = -10$, $l_1 = \beta_1 - \alpha_1$, $l_2 = \beta_2 - \alpha_2$ (curves *3*).

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